MOTION AND DISSOLUTION OF A DROP IN A CURRENT-CARRYING LIQUID

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Flow of a liquid carrying an electric current around a nonconducting drop is considered. The results are used for determining the rate of mass exchange between the drop and the surrounding liquid.

If a current-carrying liquid surrounds a body whose electrical conductivity is different from that of the liquid, the nonuniformities which arise in the density distribution of the electric current produce a nonpotential field of electromagnetic forces. These forces alter the character of the flow near the body and thereby affect the rate of mass exchange between the body and the current-carrying liquid.

Consider the low-velocity flow produced by electromagnetic forces near a stationary drop. Assuming that the drop retains its shape and neglecting the inertial terms, we write the equation of motion (the coordinate system is shown in Fig. 1),

$$-\operatorname{grad} P-\mu \operatorname{rot}\operatorname{rot} U+I\times B=0 \tag{1}$$

or

$$\mu \operatorname{rot}\operatorname{rot}\operatorname{rot} U=\operatorname{rot}(I\times B). \tag{2}$$

The expressions for I × B and rot(I × B) in the induction-free approximation are given in [1]. They have the following form:

$$I\times B=-\frac{1}{2}\mu_e I_0^2 r\sin\theta\left[1-\left(\frac{a}{r}\right)^3\right]\left\{i_r\frac{1}{2}\sin\theta\left[2+\left(\frac{a}{r}\right)^3\right]+i_\theta\cos\theta\left[1-\left(\frac{a}{r}\right)^3\right]\right\}; \tag{3}$$

$$\operatorname{rot}(I\times B)=-i_\varphi\frac{3}{2}\mu_e I_0^2\sin\theta\cos\theta\left(\frac{a}{r}\right)^3\left[1-\left(\frac{a}{r}\right)^3\right]. \tag{4}$$

We introduce a stream function ψ, such that

$$u_r=\frac{1}{r^2\sin\theta}\cdot\frac{\partial\psi}{\partial\theta};\quad u_\theta=-\frac{1}{r\sin\theta}\cdot\frac{\partial\psi}{\partial r}. \tag{5}$$

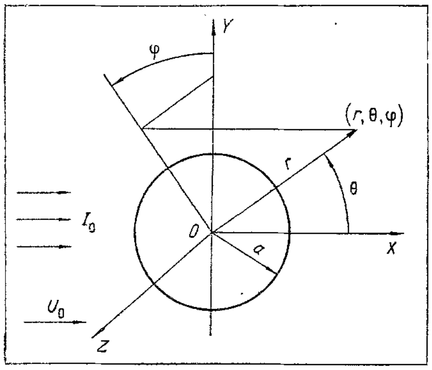

Fig. 1. Schematic presentation of the problem.

By substituting (4) and (5) in (2), we obtain the equation determining the stream function for the region outside the drop:

$$\left[\frac{\partial^2}{\partial r^2}+\frac{\sin\theta}{r^2}\cdot\frac{\partial}{\partial\theta}\left(\frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial\theta}\right)\right]^2\psi$$

$$=-\frac{3}{2}\frac{\mu_e}{\mu}I_0^2 r\sin^2\theta\cos\theta\left(\frac{a}{r}\right)^3\left[1-\left(\frac{a}{r}\right)^3\right]. \tag{6}$$

For the region inside the drop,we correspondingly have

$$\left[\frac{\partial^2}{\partial r^2}+\frac{\sin\theta}{r^2}\cdot\frac{\partial}{\partial\theta}\left(\frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial\theta}\right)\right]^2\psi'=0. \tag{7}$$

Before solving Eqs. (6) and (7), we must determine the form of the supposed solution. This can be done by investigating the character

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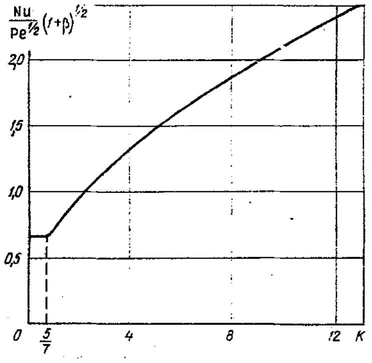

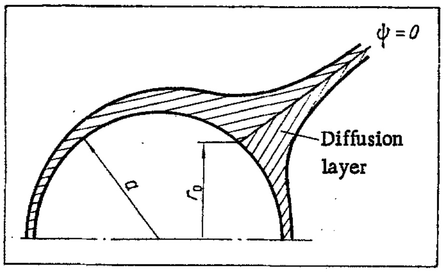

Fig. 2. Dimensionless number relationships determining the effect of an electric current on the mass-exchange rate.

Fig. 3. Thickness distribution of the diffusion layer over the drop surface for K > 5/7.

of the electromagnetic forces causing the motion of the liquid. It is evident from Eq. (3) that the field of electromagnetic forces is axisymmetric with respect to the X axis and symmetric with respect to the YOZ plane. Since the liquid flow possesses a similar symmetry, the stream function should be sought in the following form:

$$\psi = R(r)\sin^2\theta\cos\theta. \tag{8}$$

By substituting (8) in (6) and assuming that $R(r) = \text{const}\, r^n$, we obtain

$$\psi = -\frac{\mu_e I_0^2 a^5}{16\mu}\left[\frac{a}{r} + \left(\frac{r}{a}\right)^2 + C_1\left(\frac{a}{r}\right)^2 + C_2\right]\sin^2\theta\cos\theta \tag{9}$$

outside the drop, and

$$\psi' = \left[C_1'\left(\frac{r}{a}\right)^3 + C_2'\left(\frac{r}{a}\right)^5\right]\sin^2\theta\cos\theta \tag{10}$$

inside the drop. It has been taken into account here that the velocity must be bounded. For determining the constants, we have for $r = a$

$$u_\theta' = u_\theta;\quad \psi' = \psi = 0;$$

$$\mu'\left(\frac{1}{r}\cdot\frac{\partial u_r'}{\partial\theta} + \frac{\partial u_\theta'}{\partial r} - \frac{u_\theta'}{r}\right) = \mu\left(\frac{1}{r}\cdot\frac{\partial u_r}{\partial\theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right). \tag{11}$$

By using conditions (11), we obtain for the stream function outside the drop

$$\psi = -\frac{\mu_e I_0^2 a^5}{16\mu}\left[\frac{a}{r} + \left(\frac{r}{a}\right)^2 + \frac{5\beta-2}{10(\beta+1)}\left(\frac{a}{r}\right)^2 - \frac{25\beta+18}{10(\beta+1)}\right]\sin^2\theta\cos\theta$$

and for the stream function inside the drop

$$\psi' = \frac{7}{160}\cdot\frac{\mu_e I_0^2 a^5}{\mu}\cdot\frac{1}{1+\beta}\left[\left(\frac{r}{a}\right)^3 - \left(\frac{r}{a}\right)^5\right]\sin^2\theta\cos\theta,$$

where $\beta = \mu'/\mu$.

If $\beta \to 0$ (solid sphere), then

$$\psi \to -\frac{\mu_e I_0^2 a^5}{32\mu}\left[2\left(\frac{a}{r}\right) + 2\left(\frac{r}{a}\right)^2 + \left(\frac{a}{r}\right)^2 - 5\right]\sin^2\theta\cos\theta,$$

which coincides with the result obtained in [1].

If $\beta \to 0$ (gas bubble),

$$\psi \to -\frac{\mu_e I_0^2 a^5}{16\mu}\left[\left(\frac{a}{r}\right) + \left(\frac{r}{a}\right)^2 - \frac{1}{5}\left(\frac{a}{r}\right)^2 - \frac{9}{5}\right]\sin^2\theta\cos\theta.$$

We shall now determine the velocity field around the drop in the case of superposition of two flows: the forced flow of the liquid surrounding the drop and the flow caused by electromagnetic forces. Assume that the current-carrying liquid moves at the velocity U_0 parallel to the electric current at a location remote from the drop, as is shown in Fig. 1. Since the velocity field in the absence of the electric current is known [2, 3], then, in the case of superposition of two flows under the above assumptions, the stream function is given by

$$\psi = \frac{U_0 a^2}{2}\left\{\left[\left(\frac{r}{a}\right)^2 - \frac{1}{2}\left(\frac{r}{a}\right)\frac{2+3\beta}{1+\beta} + \frac{1}{2}\left(\frac{a}{r}\right)\frac{\beta}{1+\beta}\right]\right.$$
$$\left. - K\left[\left(\frac{a}{r}\right) + \left(\frac{r}{a}\right)^2 + \frac{5\beta-2}{10(\beta+1)}\left(\frac{a}{r}\right)^2 - \frac{25\beta+18}{10(\beta+1)}\right]\cos\theta\right\}\sin^2\theta \tag{12}$$

for the region outside the drop, and

$$\psi' = -\frac{U_0 a^2}{4(1+\beta)}\left\{\left[\left(\frac{r}{a}\right)^2 - \left(\frac{r}{a}\right)^4\right] - \frac{7}{5}K\left[\left(\frac{r}{a}\right)^3 - \left(\frac{r}{a}\right)^5\right]\cos\theta\right\}\sin^2\theta$$

for the region inside the drop. Here, $K = \mu_e I_0^2 a^3 / 8\mu U_0$. Evidently, the liquid flow caused by the electric current does not affect the drag of the drop because of the symmetry relative to the YZ plane.

The solution of the hydrodynamic problem concerning the motion of an electrically nonconducting drop in a current-carrying liquid makes it now possible to solve the problem of the effect of an electric current on mass exchange under these conditions.

Consider steady-state mass transfer from the surface of the drop. Since the values of the Pe number in the liquid are sufficiently large, it can be assumed that the concentration varies within a thin diffusion layer at the surface of the drop. Therefore, we have the following system of differential equations describing the concentration of dissolved matter $c = c(r, \theta)$:

$$\begin{cases} u_r \dfrac{\partial c}{\partial r} + \dfrac{u_\theta}{r}\cdot\dfrac{\partial c}{\partial \theta} = D\dfrac{\partial^2 c}{\partial r^2}; \\ c(a,\ \theta) = c_s; \\ c(\infty,\ \theta) = c_0; \\ c(r > a,\ 0) \neq c_s; \quad c(r > a,\ \pi) \neq c_s. \end{cases} \tag{13}$$

Here, u_r and u_θ are determined by Eqs. (12) and (5). The solution of system (13) is obtained by using the method described in detail in [4]. As a result, we obtain the concentration distribution $c = c(r, \theta)$, by means of which we find the dimensionless flux of matter from the surface of the drop:

$$\mathrm{Nu} = \frac{1}{\sqrt{\pi}}\cdot\frac{\mathrm{Pe}^{1/2}}{(1+\beta)^{1/2}}\left\{\left[-\frac{1}{12}\left(\frac{5}{7K}\right)^3 + \frac{1}{2}\left(\frac{5}{7K}\right)\right.\right.$$
$$\left.\left. + \frac{1}{4}\left(\frac{7K}{5}\right) + \frac{2}{3}\right]^{1/2} + \left[-\frac{1}{12}\left(\frac{5}{7K}\right)^3 + \frac{1}{2}\left(\frac{5}{7K}\right) + \frac{1}{4}\left(\frac{7K}{5}\right) - \frac{2}{3}\right]^{1/2}\right\}. \tag{14}$$

The dependence (14) is shown in Fig. 2. If $K \gg 5/7$,

$$\mathrm{Nu} = \left(\frac{7}{5\pi}\right)^{1/2}\frac{\mathrm{Pe}^{1/2}}{(1+\beta)^{1/2}}K^{1/2}. \tag{15}$$

If $K \leq 5/7$, the electric current does not affect the mass exchange rate. For $U_0 = 0$,

$$\mathrm{Nu} = \frac{1}{2}\left(\frac{7}{5\pi}\right)^{1/2}\left(\frac{\mu_e I_0^2 a^4}{\mu\nu}\cdot\frac{1}{1+\beta}\mathrm{Pr}\right)^{1/2}. \tag{16}$$

If the viscosity of matter in the drop is negligibly low, we obtain from Eqs. (14), (15), and (16) the corresponding expressions for the diffusion flux from the surface of a gas bubble.

The thickness distribution of the diffusion layer over the surface of a drop for $K > 5/7$ (see Fig. 3) is of interest. It is seen that, in the presence of an electric current, the removal of dissolved matter occurs at the surface of a ring with the radius r_0 and not at the stern end of the drop. If K increases, r_0 tends to a, and the thickness distribution of the diffusion layer becomes symmetric with respect to the YZ plane.

The above results indicate that there is a possibility of intensifying the processes of mass exchange between a drop (gas bubble) and a current-carrying liquid.

NOTATION

X, Y, Z	are the Cartesian coordinates;
r, θ, φ	are the spherical coordinates;
i_r, i_θ, and i_φ	are the unit vectors in the r, θ, and φ directions, respectively;
a	is the drop radius;
c	is the concentration of dissolved matter;
c_0	is the concentration of dissolved matter at a location remote from the drop;
c_s	is the concentration of dissolved matter at the drop surface;
U_0	is the velocity of the liquid at a point remote from the drop;
u_r	is the radial component of the velocity;
u_θ	is the tangential component of the velocity;
I	is the electric current density;
I_0	is the electric current density at a point remote from the drop;
B	is the magnetic induction;
P	is the pressure;
μ_e	is the magnetic permeability of the liquid;
μ	is the dynamic viscosity of the liquid;
μ'	is the dynamic viscosity of matter in the drop;
$\beta = \mu'/\mu$	is the ratio of the dynamic viscosity of matter in the drop to the dynamic viscosity of the surrounding liquid;
ν	is the kinematic viscosity of the liquid;
D	is the diffusion coefficient;
k	is the mass-transfer coefficient;
ψ	is the stream function for the region outside the drop;
ψ'	is the stream function for the region inside the drop;
$Nu = k2a/D$	is the diffusion Nusselt number;
$Pr = \nu/D$	is the diffusion Prandtl number;
$Pe = U_0 2a/D$	is the diffusion Peclet number;
$Re = U_0 2a/\nu$	is the Reynolds number;
C_1, C_2, C_1', C_2'	are the constants;
$K = \mu_e I_0^2 a^3/8\mu U_0$	is a dimensionless parameter.

LITERATURE CITED

1. C. Y. Chow, Phys. Fluids, 9, 933 (1966).
2. S. Hadamard, Compt. Rend., 152, 1735 (1911).
3. W. Rybczynski, Bull. Acad. Sci. Cracovie, A40 (1911).
4. V. G. Levich, Physicochemical Hydrodynamics [in Russian], Fizmatgiz, Moscow (1959).